Algorithms, Data Structures, and Complexity Theory
Fall, 2002

S.L. Bloom

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Do all problems. No books or notes may be used.

1. (10 points) **Define** completely the classes P and NP.
2. (15 points) What is the relation between log-space reduction and polynomial time reduction? Prove your answer.
3. (15 points) Recently, E.W. Dijkstra, one of the founders of Theoretical Computer Science, died. Give an account of his "shortest path algorithm".
4. (20 points). Analyze the complexity of the following problem. Suppose that $G = (V, E, \ell)$ is a **directed graph** whose edges are labeled by either 0 or 1: $\ell : E \to \{0, 1\}$. If $v_1, \ldots, v_{n+1}$ is a path $v_1 \rightsquigarrow v_{n+1}$ in G, the **label of the path** is the binary word formed by concatenating the labels of each edge on the path.

   Instance: Such a finite, edge labeled directed graph $G = (V, E, \ell)$ such that for each vertex u there are exactly two edges (u, v) and (u, w) with the lable of (u, v) being 0 and the label of (u, w) being 1.

   Question: Is there a vertex $v \in V$ and paths $v \rightsquigarrow v$, one labeled $0x$ and the other labeled $1y$, for some binary words x, y?
5. (10 points) Define: a problem is in **PSPACE**, and is a complete PSPACE problem.
6. (10 points) Give an example of a complete PSPACE problem.
7. (20 points) Suppose that there is an NP-complete problem that has a deterministic solution that runs in time $n^{\log_2 n}$. (The function $f(n) = n^{\log_2 n}$ is strictly between the polynomial functions n^k and the exponential functions k^n.) What can then be said about the running time of any problem in NP?