Qualifying Exam: Fall 2003
Data structures, algorithms, and complexity

Problem 1:

a). (5 points): Define precisely what $f(n) = O(g(n))$ means.

b). (15 points): Fill in the rest of following table, where each entry should be O, Ω, or Θ according to whether the row function is O, Ω, or Θ of the column function. If more than one is true, you should put the strongest result possible.

	n^5	$(\sqrt{2})^{\log n}$	2^n
$\log(n)$	O		
2^{n+1}			
$n^{\log n}$			
$\sqrt{n}$			

Problem 2:

a). (5 points): Draw one binary search tree of height 2 (where height refers to counting edges), and one of height 3, on the elements $\{1, 3, 4, 6, 8, 10, 12\}$.

b). (15 points): Describe the standard algorithms in a binary search tree for searching, finding the minimum, and inserting an element. What is the running time of your algorithm as a function of the height of the tree? as a function of the number of elements?

Problem 3:

a). (5 points): Define what it means for a decision problem to be NP-complete.

b). (15 points): Define the vertex cover decision problem, and show that it is NP-complete. You may use without proof the fact that 3SAT is NP-complete.