Qualifying Exam: Fall 2005
Algorithms

Do all problems. No books or notes may be used. Points will be given based on the correctness, completeness, and style of your answers. Each problem is worth 20 points.

Problem 1:

a). (5 points): Define precisely what $f(n) = O(g(n))$, $f(n) = \Omega(g(n))$, and $f(n) = \Theta(g(n))$ mean.

b). (15 points): Let $f(n)$ and $g(n)$ be nonnegative functions. Is the following equation true or false?

$$\max(f(n), g(n)) = \Theta(f(n) + g(n))$$

Either give a detailed proof that it is true (using the definition you gave in part a), or give a counterexample.

Problem 2: Give an algorithm that given a list of n elements, finds the second smallest element using $n + \lceil \log n \rceil - 2$ comparisons in the worst case. (The log is base 2. You may assume the n elements are distinct.)

Problem 3:

a). (5 points): Define **P** and **NP**, and define what it means to be **NP**-complete.

b). (15 points): Recall that SAT is defined to be the problem: given a Boolean expression in CNF form, is it satisfiable? 4-SAT is defined to be the problem: given a Boolean expression in CNF form in which each clause contains exactly four literals, is it satisfiable?

Taking as a given that SAT is **NP**-complete, prove that 4-SAT is **NP**-complete.