Qualifying Exam: Fall 2006
Algorithms

Do all problems. No books or notes may be used. Points will be given based on the correctness, completeness, and style of your answers. Each problem is worth 20 points.

Problem 1:

a). (5 points): Define precisely what $f(n) = O(g(n))$, $f(n) = \Omega(g(n))$, and $f(n) = \Theta(g(n))$ mean.

b). (15 points): Fill in the rest of following table, where each entry should be O, Ω, or Θ according to whether the row function is O, Ω, or Θ of the column function. If more than one is true, you should put the strongest result possible. All logarithms are base 2 unless otherwise specified.

	$n^{(\log n)}$	$\log_{10} n$	2^n
$\log n$	O		
$2^{(n+1)}$			
$n^{\sqrt{n}}$			
3^n			

Problem 2: Binary search trees can be used to maintain a sorted list of elements for efficient sorting and searching. Define the properties of a binary search tree.

Describe efficient algorithm to insert elements into a binary search tree (starting from an empty tree), to return the minimum element of the search tree, and to delete an element from the search tree. (For the deletion algorithm, you can give your answer primarily in terms of the multiple high-level objectives that the algorithm needs to meet).

Describe briefly why your algorithms are correct and give the running times of your algorithms. You can assume all elements that will be inserted into the tree are unique.

Problem 3:

a). (5 points): Define **P** and **NP**, and define what it means to be **NP**-complete.

b). (15 points): The VERTEX COVER (VC) problem is defined as follows:

Instance: A graph $G = (V, E)$ and a positive integer $K \leq V$.
Question: Is there a vertex cover of size K or less for G?

and the CLIQUE problem is defined as follows:

Instance: A graph $G = (V, E)$ and a positive integer $J \leq V$.
Question: Does G contain a clique of size J or more?

Using the fact that VC is **NP**-complete, show that CLIQUE is **NP**-complete.