Qualifying Examination Fall 2007
Algorithms

Do all problems. No books or notes may be used. Your grade depends on the correctness, completeness and style of your answer.

1. (10 points) Define: The 3SAT problem
2. (10 points) Define: A problem is **NP complete**
3. (20 points) What is the complexity of the following algorithm to determine whether a positive integer n is a prime? Justify your answer. (Assume you can compute $n \mod i$ in one step.)

```
if (n=1) return no;
if (n=2) return yes;
else
i=2;
while (i*i < n)
   {
   if (n mod i) = 0
        return no;
   else i=i+1;
   }
return yes.
```

4. (20 points) Suppose that $a_1 < a_2 < \ldots < a_n$ and $b_1 < b_2 < \ldots < b_n$ are integers.

   Find the most efficient algorithm you can to determine whether there is some i, j with $a_i = b_j$. The more efficient the algorithm, the more credit you get.

5. (20 points) What is the complexity of the following problem?

   INSTANCE: a finite graph $G = (V, E)$, two vertices $u, v \in V$ and an integer k.
   QUESTION: is there a path $u \rightsquigarrow v$ in G with no repeated vertices and **at most** k edges?

   Justify your answer.

6. (20 points) What is the complexity of the following problem?

   INSTANCE: a finite graph $G = (V, E)$, two vertices $u, v \in V$ and an integer k.
   QUESTION: is there a path $u \rightsquigarrow v$ in G with no repeated vertices and **at least** k edges?

   Justify your answer.