

# Algorithms, Data Structures, and Complexity Theory

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Do all problems.

1. Define completely the classes P and NP.
2. What is a polynomial time reduction?
3. What is an NP complete problem?
4. Give a LINEAR TIME algorithm to solve the following problem. Given a finite graph  $G = (V, E)$ , determine whether  $G$  contains a cycle of odd length.

Prove that your algorithm is indeed linear time.

5. A graph  $G = (V, E)$  has a  $K$ -coloring if there is an assignment of an integer in the range  $1, \dots, K$  to each vertex such that if  $(u, v) \in E$  then  $u, v$  are not assigned the same integer.

Determine the complexity of the following decision problem, and prove your answer.

Input: a finite graph  $G = (V, E)$  and a positive integer  $K$ .

Question: does  $G$  have a  $K$  coloring?