Qualifying Exam: Spring 2005
Data structures, algorithms, and complexity

Do all problems. No books or notes may be used. Points will be given based on the correctness, completeness, and style of your answers. Each problem is worth 20 points.

Problem 1:

a). (5 points): Define precisely what $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$ means.

b). (15 points): Is the following statement true or false? For every pair f, g of nonnegative functions (i.e. whose domains and ranges are nonnegative integers), either $f(n) = O(g(n))$ or $f(n) = \Omega(f(n))$ (or both).

Either prove the statement is true (using the definition you gave in part a), or give a counterexample.

Problem 2:

Give an algorithm to perform depth first search on an undirected graph, and analyze the performance of your algorithm.

Problem 3:

a). (4 points): Define DNF Boolean expressions and CNF Boolean expressions.

b). (8 points): Show that any Boolean expression in DNF form can be written as an equivalent expression in CNF form, and vice versa.

c). (8 points): We define DNF-SAT and CNF-SAT to be versions of the SAT problem taking a DNF Boolean expression or a CNF Boolean expression, respectively. Does part b imply that DNF-SAT $\in \mathbf{P}$ iff CNF-SAT $\in \mathbf{P}$? Why or why not?