

CS 385
Homework Assignment 1
Due by Fri, Sep 12, 1pm.

Prof. Dietrich

Solutions

1. The sum of the 64 squares is $2^0 + 2^1 + 2^2 + \dots + 2^{63} = 2^{64} - 1$.
2. The sum is

$$S(n) = 1^2 + 2^2 + 5^2 + 10^2 + \dots + ((n-1)^2 + 1)^2.$$

To get a lower bound, we use the fact that $i^2 < i^2 + 1$, so that

$$\begin{aligned} S(n) &> \sum_{i=0}^{n-1} (i^2)^2 \\ &= \sum_{i=0}^{n-1} i^4. \end{aligned}$$

By looking at the area under the curve $f(x) = x^4$, between $x = 0$ and $x = n - 1$, we see

$$\int_{x=0}^{n-1} x^4 dx < S(n) < \int_{x=0}^n x^4 dx,$$

so that

$$\frac{(n-1)^5}{5} < S(n) < \frac{n^5}{5},$$

showing $S(n) = \Theta(n^5)$.

3. Find the order of growth of $S(n) = \sum_{i=0}^{n-1} \sum_{j=0}^{i-1} (i+j)$.

For a fixed i ,

$$\begin{aligned}\sum_{j=0}^{i-1} (i+j) &= (i+0) + (i+1) + \dots + (i+(i-1)) \\ &= i^2 + i(i-1)/2.\end{aligned}$$

Thus,

$$\begin{aligned}S(n) &= \sum_{i=0}^{n-1} (i^2 + i(i-1)/2) \\ &< 2 \sum_{i=0}^{n-1} i^2 \\ &< 2n^3,\end{aligned}$$

say. Also,

$$\begin{aligned}S(n) &> (1/2) \sum_{i=1}^{n-1} (i-1)^2 \\ &> (1/12)n^3.\end{aligned}$$

Thus, $S(n) = \Theta(n^3)$.

4. The guess problem.

```
#include <stdio.h>
#include <stdlib.h>
#include <time.h>

#define UB 10000      /* change if you want */

int findmax( int A[], int size);

int main(int argc, char ** argv)
{
    if (argc != 3)
    {
        printf("Usage: guess S NR\n");
        printf("to run the guess function on arrays of size S NR times\n");
        return 1;
    }
    int size=atoi( argv[1] );
    int NR=atoi( argv[2] );
```

```
int * A= malloc( size*sizeof( int ));

double g = 0.0;
int count = 0;
while (count < NR)
{
    int seed=time(0);
    int i;
    srand( seed );
    for (i=0; i<size; ++i)
        A[i]=(rand() % UB);

    g= ( count * g + findmax(A, size))/(count+1);
    count++;
}
printf("avg no of times guess changed was %f\n", g);

return 0;
}

int findmax( int A[], int S)
{
    int i=1;
    int g=A[0];
    int count=0;
    while (i<S)
    {
        if (g<A[i])
        {
            g=A[i];
            count++;
        }
        i++;
    }
    return count;
}
```