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Gadget Testing Drop the first gadget from floors $\lceil\sqrt{n}\rceil, 2\lceil\sqrt{n}\rceil$, and so on until either the floor $i\lceil\sqrt{n}\rceil$ a drop from which makes the gadget malfunction is reached or no such floor in this sequence is encountered before the top of the building is reached. In the former case, the floor to be found is higher than $(i-1)\lceil\sqrt{n}\rceil$ and lower than $i\lceil\sqrt{n}\rceil$. So, drop the second gadget from floors $(i-1)\lceil\sqrt{n}\rceil + 1, (i-1)\lceil\sqrt{n}\rceil + 2$, and so on until the first floor a drop from which makes the gadget malfunction is reached. The floor immediately preceding that floor is the floor in question. If no drop in the first-pass sequence resulted in the gadgets failure, the floor in question is higher than $i\lceil\sqrt{n}\rceil$, the last tried floor of that sequence. Hence, continue the successive examination of floors $i\lceil\sqrt{n}\rceil + 1, i\lceil\sqrt{n}\rceil + 2$, and so on until either a failure is registered or the last floor is reached. The number of times the two gadgets are dropped doesn't exceed $i\lceil\sqrt{n}\rceil + \lceil\sqrt{n}\rceil$, which puts it in $O(\sqrt{n})$.