1. Classical multiplication method for polynomials of degree less than $n$ requires $2n^2 - 2n + 1$ ring operations (Section 2.3). For which values of $n = 2^k$ classical method takes more operations than $9 \cdot 3^k - 8 \cdot 2^k$ for Karatsuba’s algorithm.

2. Determine all $n \in \mathbb{N}, n > 0$ for which $F_{19}^\times$ contains a primitive $n$th root of unity. For all such $n$, list all primitive $n$th roots of unity.

3. Let $R$ be a ring, $n \in \mathbb{N}, n \geq 2, w \in R$ a primitive $n$th root of unity, and $v \in R$ with $v^2 = w$. Under what conditions is $v$ a primitive $2n$th root of unity?

4. Let $F = F_{29}$
   (a) Find a primitive 4th root of unity $w$ and compute its inverse $w^{-1} \in F$.
   (b) Find the matrices for $DFT_w$ and $DFT_{w^{-1}}$ and check that their product is $4I$.

5. Let $F = F_{17}$ and $f = 5x^3 + 3x^2 - 4x + 3, g = 2x^3 - 5x^2 + 7x - 2 \in F[x]$
   (a) Show that $w = 2$ is a primitive 8th root of unity in $F$ and compute $w^{-1} = 2^{-1} \mod 17$
   (b) Compute $h = f \cdot g \in F[x]$
   (c) Compute $\alpha_i = f(w^i), \beta_i = g(w^i)$ and $\gamma_i = y(w^i), i = 0, \ldots, 7$. Compare $\gamma_i$ and $h(w^i)$
   (d) Trace FFT algorithm which computes $DFT_w(f)$.

This homework is due by October 9.