
October 31, 2008

1. Let $\mathbb{F}_q$ be a finite field with $q$ elements.
   (a) Show that that every element $a \in \mathbb{F}_q^\times$ different from $\pm 1$ has $a^{-1} \neq a$.
   (b) Prove Wilson’s theorem:
   $\prod_{a \in \mathbb{F}_p^\times} a = -1$.

2. Use Mathematica to compute distinct degree decomposition of the squarefree polynomial
   \[ x^{17} + 2x^{15} + 4x^{13} + 2x^{12} + 2x^{10} + 3x^9 + 4x^8 + 4x^4 + 3x^3 + 2x^2 + 4x \in \mathbb{F}_5[x] \]
   Tell from the output only how many irreducible factors of degree $i$ the polynomial $f$ has, for all $i$.
   Note: you have to perform steps of the distinct-degree factorization algorithm.

3. Let $\mathbb{F}_q$ be a field and $a, b \in \mathbb{F}_q^\times$ two nonsquares. Prove that $ab$ is a square.

4. Prove or disprove:
   - The polynomial $x^{1000} + 2 \in \mathbb{F}_5[x]$ is squarefree.
   - Let $F$ be a field and $f, g \in F[x]$. Then the squarefree part of $fg$ is the product of squarefree parts of $f$ and of $g$.

5. Use Mathematica to compute the squarefree decomposition of the polynomials given in exercise 14.29 (page 415) over field $\mathbb{F}_3$.

   This homework is due by November 6.