CS 558: Computer Vision
5th Set of Notes

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Overview

• Hough Transform
• Template Matching
• Image Alignment
  – Based on slides by S. Lazebnik, K. Grauman and D. Hoiem
Fitting: The Hough transform

Slides based on S. Lazebnik’s and K. Grauman’s slides
Voting schemes

• Let each feature vote for all the models that are compatible with it
• Hopefully the noise features will not vote consistently for any single model
Hough transform

- An early type of voting scheme
- General outline:
  - Discretize *parameter space* into bins
  - For each feature point in the image, put a vote in every bin in the parameter space that could have generated this point
  - Find bins that have the most votes

Parameter space representation

- A line in the image corresponds to a point in Hough space

Image space

\[ y = m_0 x + b_0 \]

Hough parameter space

Source: S. Seitz
Parameter space representation

- What does a point \((x_0, y_0)\) in the image space map to in the Hough space?
Parameter space representation

- What does a point \((x_0, y_0)\) in the image space map to in the Hough space?
  - Answer: the solutions of \(b = -x_0m + y_0\)
  - This is a line in Hough space

![Diagram showing the relationship between image space and Hough parameter space](image)
Parameter space representation

- Where is the line that contains both \((x_0, y_0)\) and \((x_1, y_1)\)?

\[
b = -x_1 m + y_1
\]
Parameter space representation

- Where is the line that contains both \((x_0, y_0)\) and \((x_1, y_1)\)?

  - It is the intersection of the lines \(b = -x_0m + y_0\) and \(b = -x_1m + y_1\)
Parameter space representation

- Where is the line that contains both \((x_0, y_0)\) and \((x_1, y_1)\)?
  - It is the intersection of the lines \(b = -x_0m + y_0\) and \(b = -x_1m + y_1\)
Parameter space representation

- Problems with the (m,b) space:
  - Unbounded parameter domains
  - Vertical lines require infinite m

- Alternative: *polar representation*

\[ x \cos \theta + y \sin \theta = \rho \]

Each point \((x,y)\) will add a sinusoid in the \((\theta, \rho)\) parameter space.
Algorithm outline

- Initialize accumulator $H$ to all zeros
- For each feature point $(x,y)$ in the image
  - For $\theta = 0$ to $180$
    - $\rho = x \cos \theta + y \sin \theta$
    - $H(\theta, \rho) = H(\theta, \rho) + 1$
  - end
- end
- Find the value(s) of $(\theta, \rho)$ where $H(\theta, \rho)$ is a local maximum
  - The detected line in the image is given by $\rho = x \cos \theta + y \sin \theta$
Basic illustration

features

votes
A more complicated image

http://ostatic.com/files/images/ss_hough.jpg
Original image

Canny edges

Vote space and top peaks

Kristen Grauman
Effect of noise

- Peak gets fuzzy and hard to locate
Effect of noise

- Number of votes for a line of 20 points with increasing noise:
Random points

- Uniform noise can lead to spurious peaks in the array
Random points

- As the level of uniform noise increases, the maximum number of votes increases too:
Dealing with noise

• Choose a good grid / discretization
  – **Too coarse:** large vote counts obtained when too many different lines correspond to a single bucket
  – **Too fine:** miss lines because some points that are not exactly collinear cast votes for different buckets

• Increment neighboring bins (smoothing in accumulator array)

• Try to get rid of irrelevant features
  – E.g., take only edge points with significant gradient magnitude
Incorporating image gradients

- Recall: when we detect an edge point, we also know its gradient direction.
- But this means that the line is uniquely determined!

- Modified Hough transform:
  - For each edge point \((x, y)\)
    \[
    \begin{align*}
    \theta &= \text{gradient orientation at } (x, y) \\
    \rho &= x \cos \theta + y \sin \theta \\
    H(\theta, \rho) &= H(\theta, \rho) + 1
    \end{align*}
    \]
    \end{align*}
Hough transform for circles

• How many dimensions will the parameter space have?
• Given an unoriented edge point, what are all possible bins that it can vote for?
• What about an oriented edge point?
Hough transform for circles

- Circle: center \((a,b)\) and radius \(r\)
  
  \[
  (x_i - a)^2 + (y_i - b)^2 = r^2
  \]

- For a fixed radius \(r\), unknown gradient direction
Hough transform for circles

- Circle: center \((a,b)\) and radius \(r\)

\[(x_i - a)^2 + (y_i - b)^2 = r^2\]

- For a fixed radius \(r\), unknown gradient direction

Intersection: most votes for center occur here.
Hough transform for circles

• Circle: center \((a,b)\) and radius \(r\)

\[
(x_i - a)^2 + (y_i - b)^2 = r^2
\]

• For an unknown radius \(r\), unknown gradient direction
Hough transform for circles

- Circle: center \((a, b)\) and radius \(r\)

\[ (x_i - a)^2 + (y_i - b)^2 = r^2 \]

- For an unknown radius \(r\), unknown gradient direction

Kristen Grauman
Hough transform for circles

- For an unknown radius $r$, known gradient direction
Hough transform for circles

For every edge pixel \((x,y)\) :
  For each possible radius value \(r\):
    For each possible gradient direction \(\theta\):
      // or use estimated gradient at \((x,y)\)
      \(a = x - r \cos(\theta)\) // column
      \(b = y + r \sin(\theta)\) // row
      \(H[a,b,r] += 1\)
  
end
end

Time complexity per edgel?

Kristen Grauman
Example: detecting circles with Hough

Note: a different Hough transform (with separate accumulators) was used for each circle radius (quarters vs. penny).
Example: detecting circles with Hough

Combined detections

Edges

Votes: Quarter

Coin finding sample images from: Vivek Kwatra
Example: iris detection

- Hemerson Pistori and Eduardo Rocha Costa

Kristen Grauman
Generalized Hough transform

- We want to find a template defined by its reference point (center) and several distinct types of landmark points in stable spatial configuration.
Generalized Hough transform

- Template representation: for each type of landmark point, store all possible displacement vectors towards the center
Generalized Hough transform

• Detecting the template:
  – For each feature in a new image, look up that feature type in the model and vote for the possible center locations associated with that type in the model
Application in recognition

• Index displacements by “visual codeword”

B. Leibe, A. Leonardis, and B. Schiele, Combined Object Categorization and Segmentation with an Implicit Shape Model, ECCV Workshop on Statistical Learning in Computer Vision 2004
Application in recognition

• Index displacements by “visual codeword”
Voting: practical tips

• Minimize irrelevant tokens first
• Choose a good grid / discretization

Too fine ? Too coarse

• Vote for neighbors, also (smoothing in accumulator array)
• Use direction of edge to reduce parameters by 1
• To read back which points voted for “winning” peaks, keep tags on the votes.
Hough transform: Discussion

• Pros
  – All points processed independently
  – Can deal with occlusion and gaps
  – Can detect multiple instances of a model
  – Some robustness to noise: noise points unlikely to contribute consistently to any single bin

• Cons
  – Complexity of search time increases exponentially with the number of model parameters
  – Non-target shapes can produce spurious peaks in parameter space
  – It’s hard to pick a good grid size
Fitting Algorithm Summary

• Least Squares Fit
  – closed form solution
  – robust to noise
  – not robust to outliers

• Robust Least Squares
  – improves robustness to noise
  – requires iterative optimization

• Hough transform
  – robust to noise and outliers
  – can fit multiple models
  – only works for a few parameters (1-4 typically)

• RANSAC
  – robust to noise and outliers
  – works with a moderate number of parameters (e.g., 1-8)
Example: solving for translation

Given matched points in \{A\} and \{B\}, estimate the translation of the object

$$
\begin{bmatrix}
    x_i^B \\
    y_i^B
\end{bmatrix} = \begin{bmatrix}
    x_i^A \\
    y_i^A
\end{bmatrix} + \begin{bmatrix}
    t_x \\
    t_y
\end{bmatrix}
$$
Example: solving for translation

Least squares solution
1. Write down objective function
2. Derived solution
   a) Compute derivative
   b) Compute solution
3. Computational solution
   a) Write in form $Ax = b$
   b) Solve using pseudo-inverse or eigenvalue decomposition

\[
\begin{bmatrix}
  x_i^B \\
  y_i^B
\end{bmatrix} = \begin{bmatrix}
  x_i^A \\
  y_i^A
\end{bmatrix} + \begin{bmatrix}
  t_x \\
  t_y
\end{bmatrix}
\]

\[
\begin{bmatrix}
  1 & 0 \\
  0 & 1 \\
  \vdots & \vdots \\
  1 & 0
\end{bmatrix} \begin{bmatrix}
  t_x \\
  t_y
\end{bmatrix} = \begin{bmatrix}
  x_1^B - x_1^A \\
  y_1^B - y_1^A \\
  \vdots \\
  x_n^B - x_n^A \\
  y_n^B - y_n^A
\end{bmatrix}
\]
Example: solving for translation

Problem: outliers

RANSAC solution
1. Sample a set of matching points (1 pair)
2. Solve for transformation parameters
3. Score parameters with number of inliers
4. Repeat steps 1-3 N times
Example: solving for translation

Problem: outliers, multiple objects, and/or many-to-one matches

Hough transform solution

1. Initialize a grid of parameter values
2. Each matched pair casts a vote for consistent values
3. Find the parameters with the most votes
4. Solve using least squares with inliers

\[
\begin{bmatrix}
  x_i^B \\
  y_i^B
\end{bmatrix} = \begin{bmatrix}
  x_i^A \\
  y_i^A
\end{bmatrix} + \begin{bmatrix}
  t_x \\
  t_y
\end{bmatrix}
\]
Template Matching

Slides based on D. Hoiem’s slides
Template matching

• Goal: find 🕍 in image

• Main challenge: What is a good similarity or distance measure between two patches?
  – Filtering
  – Zero-mean filtering
  – Sum of Squares Difference
  – Normalized Cross Correlation
Matching with filters

- **Goal:** find \(\begin{array}{c}
\text{in image}
\end{array}\)
- **Method 0:** filter the image with eye patch

\[ h[m, n] = \sum_{k,l} g[k,l] \cdot f[m+k, n+l] \]

What went wrong?

Input  
Filtered Image  

\(f = \text{image}\)  
\(g = \text{filter}\)
Matching with filters

- Goal: find in image
- Method 1: filter the image with zero-mean eye

\[ h[m,n] = \sum_{k,l} (g[k,l] - \bar{g}) (f[m+k,n+l]) \]

where \( \bar{g} \) is the mean of template g.

Input | Filtered Image (scaled) | Thresholded Image

True detections | False detections
SSD

- Goal: find in image
- Method 2: SSD

\[ h[m,n] = \sum_{k,l} (g[k,l] - f[m+k,n+l])^2 \]
SSD

- Goal: find in image
- Method 2: SSD

$$h[m,n] = \sum_{k,l} (g[k,l] - f[m + k, n + l])^2$$

What’s the potential downside of SSD?
NCC

- **Goal:** find in image
- **Method 3:** Normalized cross-correlation

\[
\begin{align*}
    h[m, n] &= \frac{\sum_{k,l} (g[k, l] - \bar{g})(f[m + k, n + l] - \bar{f}_{m,n})}{\left(\sum_{k,l} (g[k, l] - \bar{g})^2 \sum_{k,l} (f[m + k, n + l] - \bar{f}_{m,n})^2\right)^{0.5}}
\end{align*}
\]

**Matlab:** `normxcorr2(template, im)`
NCC

- Goal: find in image
- Method 3: Normalized cross-correlation
NCC

- Goal: find \( \square \) in image
- Method 3: Normalized cross-correlation
Q: What is the best method to use?

A: Depends

- Zero-mean filter: fastest but not a great matcher
- SSD: next fastest, sensitive to overall intensity
- Normalized cross-correlation: slowest, invariant to local average intensity and contrast
Q: What if we want to find larger or smaller eyes?

A: Image Pyramid
Review of Sampling

Image → Gaussian Filter → Low-Pass Filtered Image → Sample → Low-Res Image
Gaussian pyramid

Source: D. Forsyth
Template Matching with Image Pyramids

Input: Image, Template
1. Match template at current scale
2. Downsample image
   - In practice, scale step of 1.1 to 1.2
3. Repeat 1-2 until image is very small
4. Take responses above some threshold, perhaps with non-maxima suppression
Laplacian filter

unit impulse

Gaussian

Laplacian of Gaussian
Laplacian pyramid

Source: D. Forsyth
Computing Gaussian/Laplacian Pyramid
Major uses of image pyramids

• Compression

• Object detection
  – Scale search
  – Features

• Detecting stable interest points

• Registration
  – Course-to-fine
Alignment

Slides based on D. Hoiem’s and S. Lazebnik’s slides
Alignment

• Alignment: find parameters of model that maps one set of points to another

• Typically want to solve for a global transformation that accounts for most true correspondences

• Difficulties
  – Noise (perturbation around true features, matches, etc.)
  – Outliers
  – Many-to-one matches or multiple objects
Parametric (global) warping

Transformation $T$ is a coordinate change

$p' = T(p)$

What does it mean that $T$ is global?
- Is the same for any point $p$
- Can be described by just a few numbers (parameters)

For linear transformations, we can represent $T$ as a matrix

$p' = Tp$

$$
\begin{bmatrix}
  x' \\
  y'
\end{bmatrix} = \begin{bmatrix}
  x \\
  y
\end{bmatrix}
$$
Common transformations

original

Transformed

- translation
- rotation
- aspect

- affine
- perspective

Slide credit (next few slides): A. Efros and/or S. Seitz
Scaling

- **Scaling** a coordinate means multiplying each of its components by a scalar.
- **Uniform scaling** means this scalar is the same for all components:
Non-uniform scaling: different scalars per component:

\[ X \times 2, \quad Y \times 0.5 \]
Scaling

• Scaling operation:

\[ x' = ax \]
\[ y' = by \]

• Or, in matrix form:

\[
\begin{bmatrix}
  x' \\
  y'
\end{bmatrix} =
\begin{bmatrix}
  a & 0 \\
  0 & b
\end{bmatrix}
\begin{bmatrix}
  x \\
  y
\end{bmatrix}
\]

scaling matrix \( S \)
2-D Rotation

\[(x', y') = (x \cos(\theta) - y \sin(\theta), x \sin(\theta) + y \cos(\theta))\]
2-D Rotation

Polar coordinates…
\[ x = r \cos(\phi) \]
\[ y = r \sin(\phi) \]
\[ x' = r \cos(\phi + \theta) \]
\[ y' = r \sin(\phi + \theta) \]

Trig Identity…
\[ x' = r \cos(\phi) \cos(\theta) - r \sin(\phi) \sin(\theta) \]
\[ y' = r \sin(\phi) \cos(\theta) + r \cos(\phi) \sin(\theta) \]

Substitute…
\[ x' = x \cos(\theta) - y \sin(\theta) \]
\[ y' = x \sin(\theta) + y \cos(\theta) \]
2-D Rotation

This is easy to capture in matrix form:

\[
\begin{bmatrix}
  x' \\
  y'
\end{bmatrix} =
\begin{bmatrix}
  \cos(\theta) & -\sin(\theta) \\
  \sin(\theta) & \cos(\theta)
\end{bmatrix}
\begin{bmatrix}
  x \\
  y
\end{bmatrix}
\]

Even though \( \sin(\theta) \) and \( \cos(\theta) \) are nonlinear functions of \( \theta \),

- \( x' \) is a linear combination of \( x \) and \( y \)
- \( y' \) is a linear combination of \( x \) and \( y \)

What is the inverse transformation?

- Rotation by \(-\theta\)
- For rotation matrices \( R^{-1} = R^T \)
Basic 2D Transformations

\[
\begin{bmatrix}
x' \\
y'
\end{bmatrix} = \begin{bmatrix}
s_x & 0 \\
0 & s_y
\end{bmatrix} \begin{bmatrix}
x \\
y
\end{bmatrix}
\]

Scale

\[
\begin{bmatrix}
x' \\
y'
\end{bmatrix} = \begin{bmatrix}
1 & \alpha_x \\
\alpha_y & 1
\end{bmatrix} \begin{bmatrix}
x \\
y
\end{bmatrix}
\]

Shear

\[
\begin{bmatrix}
x' \\
y'
\end{bmatrix} = \begin{bmatrix}
\cos \Theta & -\sin \Theta \\
\sin \Theta & \cos \Theta
\end{bmatrix} \begin{bmatrix}
x \\
y
\end{bmatrix}
\]

Rotate

\[
\begin{bmatrix}
x' \\
y'
\end{bmatrix} = \begin{bmatrix}
1 & 0 & t_x \\
0 & 1 & t_y
\end{bmatrix} \begin{bmatrix}
x \\
y \\
1
\end{bmatrix}
\]

Translate

\[
\begin{bmatrix}
x' \\
y'
\end{bmatrix} = \begin{bmatrix}
a & b & c \\
d & e & f
\end{bmatrix} \begin{bmatrix}
x \\
y \\
1
\end{bmatrix}
\]

Affine

Affine is any combination of translation, scale, rotation, shearing
Affine Transformations

Affine transformations are combinations of:
- Linear transformations, and
- Translations

Properties of affine transformations:
- Lines map to lines
- Parallel lines remain parallel
- Ratios are preserved
- Closed under composition

\[
\begin{bmatrix}
x' \\
y'
\end{bmatrix} = \begin{bmatrix}
a & b & c \\
d & e & f
\end{bmatrix} \begin{bmatrix}
x \\
y \\
1
\end{bmatrix}
\]

or

\[
\begin{bmatrix}
x' \\
y' \\
1
\end{bmatrix} = \begin{bmatrix}
a & b & c \\
d & e & f \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
x \\
y \\
1
\end{bmatrix}
\]
Projective Transformations

Projective transformations are combos of

- Affine transformations, and
- Projective warps

Properties of projective transformations:

- Lines map to lines
- Parallel lines do not necessarily remain parallel
- Ratios are not preserved
- Closed under composition
- Models change of basis
- Projective matrix is defined up to a scale (8 DOF)
2D image transformations (reference table)

<table>
<thead>
<tr>
<th>Name</th>
<th>Matrix</th>
<th># D.O.F.</th>
<th>Preserves:</th>
<th>Icon</th>
</tr>
</thead>
<tbody>
<tr>
<td>translation</td>
<td>$[ I \</td>
<td>\ t ]_{2 \times 3}$</td>
<td>2</td>
<td>orientation + ⋅⋅⋅</td>
</tr>
<tr>
<td>rigid (Euclidean)</td>
<td>$[ R \</td>
<td>\ t ]_{2 \times 3}$</td>
<td>3</td>
<td>lengths + ⋅⋅⋅</td>
</tr>
<tr>
<td>similarity</td>
<td>$[ sR \</td>
<td>\ t ]_{2 \times 3}$</td>
<td>4</td>
<td>angles + ⋅⋅⋅</td>
</tr>
<tr>
<td>affine</td>
<td>$[ A ]_{2 \times 3}$</td>
<td>6</td>
<td>parallelism + ⋅⋅⋅</td>
<td>△</td>
</tr>
<tr>
<td>projective</td>
<td>$[ \tilde{H} ]_{3 \times 3}$</td>
<td>8</td>
<td>straight lines</td>
<td>□</td>
</tr>
</tbody>
</table>
Image alignment: Applications

Recognition of object instances
Image alignment: Challenges

Small degree of overlap
Intensity changes

Occlusion, clutter
Feature-based alignment

- Search sets of feature matches that agree in terms of:
  a) Local appearance
  b) Geometric configuration
Alignment as fitting

- Previously: fitting a model to features in one image

\[
\sum_{i} \text{residual}(x_i, M)
\]

Find model \( M \) that minimizes \( \sum_{i} \text{residual}(x_i, M) \)
Alignment as fitting

• Alignment: fitting a model to a transformation between pairs of features (matches) in two images

Find transformation $T$ that minimizes

$$\sum_i \text{residual}(T(x_i), x'_i)$$
Let’s start with affine transformations

- Simple fitting procedure (linear least squares)
- Approximates viewpoint changes for roughly planar objects
- Can be used to initialize fitting for more complex models
Fitting an affine transformation

• Assume we know the correspondences, how do we get the transformation?

\[
\begin{align*}
\begin{bmatrix} x'_i \\ y'_i \end{bmatrix} &= \begin{bmatrix} m_1 & m_2 \\ m_3 & m_4 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} + \begin{bmatrix} t_1 \\ t_2 \end{bmatrix} \\
\end{align*}
\]

Want to find \( M, t \) to minimize

\[
\sum_{i=1}^{n} \left\| x'_i - Mx_i - t \right\|^2
\]
Fitting an affine transformation

\[
\begin{bmatrix}
  x_i' \\
  y_i'
\end{bmatrix} = \begin{bmatrix}
  m_1 & m_2 \\
  m_3 & m_4
\end{bmatrix} \begin{bmatrix}
  x_i \\
  y_i
\end{bmatrix} + \begin{bmatrix}
  t_1 \\
  t_2
\end{bmatrix}
\]

\[
\begin{bmatrix}
  m_1 \\
  m_2 \\
  m_3 \\
  m_4 \\
  t_1 \\
  t_2
\end{bmatrix} = \begin{bmatrix}
  x_i & y_i & 0 & 0 & 1 & 0 \\
  0 & 0 & x_i & y_i & 0 & 1 \\
  \cdots
\end{bmatrix}
\]

\[
\begin{bmatrix}
  \cdots \\
  x_i' \\
  y_i'
\end{bmatrix}
\]
Fitting an affine transformation

\[
\begin{bmatrix}
  x_i & y_i & 0 & 0 & 1 & 0 \\
  0 & 0 & x_i & y_i & 0 & 1 \\
  \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
  \end{bmatrix}
\begin{bmatrix}
  m_1 \\
  m_2 \\
  m_3 \\
  m_4 \\
  t_1 \\
  t_2 \\
\end{bmatrix}
= \begin{bmatrix}
  \cdots \\
  x'_i \\
  y'_i \\
  \cdots \\
\end{bmatrix}
\]

- Linear system with six unknowns
- Each match gives us two linearly independent equations: need at least three to solve for the transformation parameters
Fitting a plane projective transformation

• Homography: plane projective transformation (transformation taking a quad to another arbitrary quad)
Homography

- The transformation between two views of a planar surface

- The transformation between images from two cameras that share the same center
Application: Panorama stitching

Source: Hartley & Zisserman
Fitting a homography

- Homogeneous coordinates (more later)

\[(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}\]

Converting to homogeneous image coordinates

\[
\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)
\]

Converting from homogeneous image coordinates

\[
\lambda \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}
\]
Fitting a homography

- Equation for homography:

\[ \lambda \begin{bmatrix} x'_i \\ y'_i \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} \]

\[ \lambda x'_i = H x_i \]

\[ x'_i \times H x_i = 0 \]

\[ \begin{bmatrix} x'_i h^T_1 x_i \\ y'_i h^T_2 x_i \\ 1 \end{bmatrix} = \begin{bmatrix} y'_i h^T_3 x_i - h^T_2 x_i \\ h^T_1 x_i - x'_i h^T_3 x_i \\ x'_i h^T_2 x_i - y'_i h^T_1 x_i \end{bmatrix} \]

\[ \begin{bmatrix} 0^T \\ x^T_i \\ -x^T_i \end{bmatrix} \begin{bmatrix} y'_i x^T_i \\ -x'_i x^T_i \end{bmatrix} = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} = 0 \]

3 equations, only 2 linearly independent
Direct linear transform

\[
\begin{bmatrix}
0^T & x_1^T & -y_1' \, x_1^T \\
x_1^T & 0^T & -x_1' \, x_1^T \\
\vdots & \vdots & \vdots \\
0^T & x_n^T & -y_n' \, x_n^T \\
x_n^T & 0^T & -x_n' \, x_n^T
\end{bmatrix}
\begin{pmatrix}
h_1 \\
h_2 \\
h_3
\end{pmatrix} = 0 \quad \text{A} \, h = 0
\]

- H has 8 degrees of freedom (9 parameters, but scale is arbitrary)
- One match gives us two linearly independent equations
- Homogeneous least squares: find h minimizing \( ||Ah||^2 \)
  - Eigenvector of \( A^T A \) corresponding to smallest eigenvalue
  - Four matches needed for a minimal solution
Robust feature-based alignment

• So far, we’ve assumed that we are given a set of “ground-truth” correspondences between the two images we want to align

• What if we don’t know the correspondences?
Robust feature-based alignment

• So far, we’ve assumed that we are given a set of “ground-truth” correspondences between the two images we want to align
• What if we don’t know the correspondences?
Robust feature-based alignment

- Extract features
Robust feature-based alignment

- Extract features
- Compute *putative matches*
Robust feature-based alignment

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Generating putative correspondences
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- Need to compare feature descriptors of local patches surrounding interest points
Feature descriptors

• Recall: feature detection and description
Feature descriptors

• Simplest descriptor: vector of raw intensity values

• How to compare two such vectors?
  – Sum of squared differences (SSD)
    \[ \text{SSD}(u, v) = \sum_i (u_i - v_i)^2 \]
    • Not invariant to intensity change

  – Normalized correlation
    \[ \rho(u, v) = \frac{(u - \bar{u}) \cdot (v - \bar{v})}{\| u - \bar{u} \| \| v - \bar{v} \|} = \frac{\sum_i (u_i - \bar{u})(v_i - \bar{v})}{\sqrt{\left( \sum_j (u_j - \bar{u})^2 \right) \left( \sum_j (v_j - \bar{v})^2 \right)}} \]
    • Invariant to affine intensity change
Disadvantage of intensity vectors as descriptors

- Small deformations can affect the matching score a lot
Slide Credits

• This set of sides contains contributions kindly made available by the following authors
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